4

Toets mechanica 07-01-'10

$\boxed{5.0}$

a $m\ddot{\vec{r}} = -mg\vec{k} - \gamma|\vec{v}|\vec{v} + q(\vec{v} \times \vec{B})$

8

b $m\ddot{x} = -\gamma|v_x|v_x + q(v_z B) = -\gamma v_x^2 + q v_x B$

X ?

1 $m\ddot{y} = -\gamma|v_y|v_y = -\gamma v_y^2$

$m\ddot{z} = -mg - \gamma|v_z|v_z + q(v_x B) = -mg - \gamma v_z^2 + q v_x B$

? s x ?

c? nee, in de formule voor de x richting komt $v_z$ voor en in de formule voor de z-richting
1 komt $v_x$ voor

Totals mechanica 07-01-'10

n kracht is conservatief als $\nabla \times \vec{F} = 0$

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ Axy^2 & Axyz & Azy^2 \end{vmatrix} = \left(\frac{d\, Azy^2}{dy} - \frac{dAxyz}{dz}\right)\vec{i} + \left(\frac{dAzy^2}{dx} - \frac{d\, Axy^2}{dz}\right)\vec{j} +$$

$$\left(\frac{dAxyz}{dx} - \frac{dAxy^2}{dy}\right)\vec{k}$$

7/7

$$= (2Azy - Axy)\vec{i} + (0-0)\vec{j} + (Ayz - 2Axy)\vec{k}$$

$$= A(2zy - xy)\vec{i} + (yz - 2xy)\vec{k} \neq 0$$ dus deze kracht is niet conservatief.

1

Toets Mechanica 7-01-'10

voor de wrijvingskracht neem ik $c \cdot m \cdot \vec{v}$

$m$ op een tijdstip is $m_0 + \int_0^t dm = m_0 + \int_0^t A\,dt = m_0 + At$

bew: $m\vec{a} = \vec{F} - cm\vec{v} = \vec{F} - c(m_0 + At)\vec{v}$

~~constante snelheid:~~ $\cancel{\vec{a} = 0}$

$\cancel{\vec{F} - c(m_0 + At) = 0}$

$\cancel{\vec{F} = c(m_0 + At)}$

$\cancel{\int m\,dv = \int_0^t \vec{F} - c(m_0 + At)\,dt}$

→ constante snelheid: $\vec{a} = 0$

$\vec{F} - c\vec{v}(m_0 + At) = 0$

$\vec{F} = c\vec{v}(m_0 + At)$

4

Toets Mechanica 7-01-'10

energiebehoud

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m (\cancel{u_1 \cdot \cos\phi})^2 + \frac{1}{2} \cdot 2m (\cancel{u_2 \cdot \cos 45^\circ})^2$$ !!

$$= \frac{1}{2} m u_1^2 \cos^2\phi + m u_2^2 \cdot \frac{1}{4} \cdot 2 = \frac{1}{2} m u_1^2 \cos^2\phi + \frac{1}{2} m u_2^2$$

impulsbehoud

$$m v_0 = m u_1 \cos\phi + 2m u_2 \cos\phi = m u_1 \cos\phi + \sqrt{2}\, m u_2$$ } x richting. y richting ?!

b impulsbehoud $v_0 = u_1 \cos\phi + \sqrt{2}\, u_2$

1

$$\sqrt{2}\, u_2 = v_0 - u_1 \cos\phi$$

$$u_2 = \frac{v_0 - u_1 \cos\phi}{\sqrt{2}} = \frac{1}{2}\sqrt{2}\,(v_0 - u_1 \cos\phi)$$

energiebehoud

$$v_0^2 = u_1^2 \cos^2\phi + u_2^2$$

$$u_1^2 \cos^2\phi = v_0^2 - u_2^2$$

$$u_1^2 = \frac{v_0^2 - u_2^2}{\cos^2\phi}$$

$$= \frac{v_0^2 - \frac{1}{2}(v_0^2 - 2v_0 u_1 \cos\phi + u_1^2 \cos^2\phi)}{\cos^2\phi}$$

$$= \frac{\frac{1}{2} v_0^2 + v_0 u_1 \cos\phi - \frac{1}{2} u_1^2 \cos^2\phi}{\cos^2\phi}$$

$$= -\frac{1}{2} u_1^2 + \frac{u_1 v_0}{\cos\phi} + \frac{v_0^2}{2\cos^2\phi}$$

$$\frac{3}{2} u_1^2 - \frac{2 u_1 v_0}{\cos\phi} - \frac{v_0^2}{2\cos^2\phi} = 0 = 3u_1^2 - \frac{4 v_0 u_1}{\cos\phi} - \frac{v_0^2}{\cos^2\phi}$$

abc-formule:

bij − zou $u_1$ negatief worden

$$u_1 = \frac{\frac{4v_0}{\cos\phi} \pm \sqrt{\frac{16 v_0^2}{\cos^2\phi} - 4 \cdot 3 \cdot \frac{-v_0^2}{\cos^2\phi}}}{6} = \frac{\frac{4 v_0}{\cos\phi} \pm \sqrt{\frac{28 v_0^2}{\cos^2\phi}}}{6}$$

$$= \frac{\frac{(4+\sqrt{28})\, v_0}{\cos\phi}}{6} = \frac{(4+\sqrt{28})\, v_0}{6\cos\phi}$$

$u_2 = \frac{1}{2}\sqrt{2}\,(v_0 - u_1\cos\phi) = \frac{1}{2}\sqrt{2}\left(v_0 - \frac{(4+\sqrt{28})v_0}{6\cos\phi}\cdot\cos\phi\right)$

$= \frac{1}{2}\sqrt{2}\left(v_0 - \frac{4+\sqrt{28}}{6}v_0\right) = \frac{1}{2}\sqrt{2}\left(1 - \frac{4+\sqrt{28}}{6}\right)v_0$

$= \left(\frac{1}{2}\sqrt{2} - \frac{\frac{1}{2}+\sqrt{14}}{6}\right)v_0 = \left(\frac{1}{2}\sqrt{2} - \frac{1}{12} + \frac{\sqrt{14}}{6}\right)v_0$

dit klopt waarschijnlijk niet, maar ik kan de fout niet

van $u_1$ en $u_2$ g...

$v_0 - \left(\frac{1}{2}\sqrt{2} - \frac{1}{12} + \ldots\right.$

$\frac{(4+\sqrt{28})\,v\ldots}{6\cos\phi}$

e) energiebehoud:

1 $0.8\left(\frac{1}{2}mv_0^2\right) = \frac{1}{2}mu_1^2\cancel{\cos\phi} + \frac{1}{2}\cdot 2mu_2^2\cdot\cancel{\cos^2 45^\circ} = \frac{1}{2}mu_1^2\cos^2\phi + \frac{1}{2}m\ldots$

$= \frac{2}{5}mv_0^2$

impulsbehoud is er nu niet. !

d) $m_1 = m \quad m_2 = 2m$

1 $\vec{v}_{cm} = \frac{\sum_i m_i \cdot v_i}{\sum_i m_i} = \frac{m\cdot v_0 + 2m\cdot 0}{m_1+m_2} = \frac{mv_0}{3m} = \frac{v_0}{3}$ voor de botsing

$= \frac{1}{3}v_0$

$= \frac{m\cdot u_1\cdot\cos\phi + 2m\cdot u_2\cdot\cos 45^\circ}{m_1+m_2} = \frac{mu_1\cos\phi + mu_2\sqrt{2}}{3m}$

$= \frac{u_1\cos\cancel{\phi + u_2\sqrt{2}}}{3}$ na de botsing.

c) $v_0 = u_1\cos\phi + u_2\sqrt{2}$ ?

0

$u_1\cos\phi = v_0 - u_2\sqrt{2}$

$\cos\phi = \frac{v_0 - u_2\sqrt{2}}{u_1}$

$\phi = \cos^{-1}\left(\frac{v_0 - u_2\sqrt{2}}{u_1}\right)$

$\cos\phi)$

van $u_1$ en $u_2$ ~~geeft~~.

$$v_0 - \frac{\left(\frac{1}{2}\sqrt{2} - \frac{1}{12} + \frac{\sqrt{14}}{6}\right) v_0 \sqrt{2}}{\frac{(4+\sqrt{28})\, v_0}{6\cos\phi}} \Bigg)$$

ik kan de fout nie

$\frac{1}{2} m u_1^2 \cos^2\phi + \frac{1}{2} m$

voor de botsing

$\frac{m u_2 \sqrt{2}}{}$

$$\left[(2m\omega v')^2 + (m\omega^2 x')^2\right]^{\frac{1}{2}} = \mu_s mg \qquad m\ddot{x} = -2m(\omega\dot{z}\cos\lambda - \omega\dot{y}\sin\lambda)$$

$$\left[4m^2\omega^2 v'^2 + m^2\omega^4 x'^2\right]^{\frac{1}{2}} = \mu_s mg \qquad \ddot{x} = -2\omega(\dot{z}\cos\lambda - \dot{y}\sin\lambda).$$

$$4m^2\omega^2 v'^2 + m^2\omega^4 x'^2 = \mu_s^2 m^2 g^2$$

$$4\omega^2 v'^2 + \omega^4 x'^2 = \mu s^2 g^2$$

$$\omega^4 x'^2 = \mu s^2 g^2 - 4\omega^2 v'^2$$

$$x'^2 = \frac{\mu s^2 g^2}{\omega^4} - \frac{4\omega^2 v'^2}{\omega^4}$$

$$x'^2 = \frac{\mu_s^2 g^2 - 4\omega^2 v'^2}{\omega^4}$$

$$x' = \frac{\left[\mu_s^2 g^2 - 4\omega^2 v'^2\right]^{\frac{1}{2}}}{\omega^2}$$

$$v_0^2 = u_1^2\cos^2\phi + u_2^2 = \frac{\left(\frac{1}{2}v_0(1+\sqrt{5})\right)^2}{\cos^2\phi}\cos^2\phi + \frac{\left(\frac{1}{2}v_0(1-\sqrt{5})\right)^2}{2}$$

$$= \frac{1}{4}v_0^2(1+\sqrt{5})^2 + \frac{\frac{1}{4}v_0^2(1-\sqrt{5})^2}{2}$$

$$\cancel{\frac{1}{4}v_0^2(1+\sqrt{5}) + \frac{1}{8}v_0^2(1+\sqrt{5})}$$

$$\frac{1}{4}v_0^2 + \frac{1}{4}\sqrt{5}v_0^2 + \frac{1}{8}v_0^2 - \frac{1}{8}\sqrt{5}v_0^2$$

$$\frac{1}{4}v_0^2(1+2\sqrt{5}+5) + \frac{\frac{1}{4}v_0^2(1-2\sqrt{5}+5)}{2}$$

$$\frac{1}{4}v_0^2 + \frac{1}{2}\sqrt{5}v_0^2 + \frac{5}{4}v_0^2 + \frac{1}{8}v_0^2 - \frac{1}{4}\sqrt{5}v_0^2 + 5$$